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A MODEL INDEPENDENT ANALYSIS OF EXPERIMENTAL BARYON MAGNETIC MOMENTS*

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ABSTRACT

Strong disagreement between experimental hyperon magnetic moments and simple model predictions is exhibited in the function $R(p,\Sigma^+,\Xi) = 3(\mu_p - \mu_{\Sigma^+})/(\mu_{\Xi^-} - \mu_{\Xi^\circ}) = 2.7 \pm 0.8$, an order of magnitude larger than the broken SU(6) prediction 0.34. This is shown to imply quenching of contributions of nonstrange quarks in strange baryon magnetic moments, relative to contributions in the nucleon. The model independent analysis includes SU(6) symmetry breaking, configuration mixing, relativistic corrections and quark-diquark correlations.

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The failure of simple constituent quark models[1] to explain[2-8] the new experimental values of Σ and Ξ magnetic moments[9,10] contrasts sharply with the remarkable success of the naive constituent quark model in describing the nucleon and Λ magnetic moments[11,12] and the hadron mass spectrum.[13,14,15] The underlying dynamical reasons for this success are not understood and it all may be accidental.[16] However, there may also be a simple explanation which awaits discovery from clues in the experimental data. This letter presents a model independent analysis of the experimental data and precisely defines the frontier between the regions where the simple model works well and where it breaks down badly.

The discrepancies in the Σ and Ξ magnetic moments are shown below to indicate a quenching of the magnetic moments of the nonstrange quarks in strange baryons. The magnetic moment of the strange quark, however, is not similarly quenched, thus ruling out scaling factors determined by hadron masses.[4] Furthermore the successful mass relations[11-16] between mesons and baryons and the baryon magnetic moment suggests that no such quenching occurs in the color magnetic moments of the nonstrange quarks which are responsible for the hyperfine splitting in the simple models. The only proposed model with the qualitative features suggested by these regularities in the data is the pion exchange model of Brown et al.[17] The two-body pion exchange contribution affects only nonstrange quarks since strange quarks are coupled to the pion. Furthermore, the pion exchange current carries electron charge but is color neutral and contributes to the electromagnetic moment without contributing to the color magnetic moment and spoiling the relations between hyperfine splittings.

Previous attempts to explain the discrepancies by using linear combinations of baryon moments which project out individual quark contributions[5,7] have not given convincing results because of the large errors in the Σ moments, particularly the Σ^-. The present analysis chooses two functions of the baryon moments which approximately project out nonstrange and strange quark contributions while avoiding large contributions from poorly known moments with large errors.

$$R(p,\Sigma^+,\Xi) = \frac{\mu_p - \mu_{\Sigma^+}}{-\frac{1}{3}\left[\mu_{\Xi^0} - \mu_{\Xi^-}\right]} = 2.7 \pm 0.8 \tag{1a}$$

$$R(\Xi,\Lambda) = \frac{\mu_{\Xi^0} + \mu_{\Xi^-}}{3\mu_\Lambda} = 1.05 \pm 0.04 \tag{1b}$$

The quantity $R(p,\Sigma^+,\Xi)$ defined by Eq.(1a) is predicted to vanish in the SU(3) symmetry limit, while the quantity $R(\Xi,\Lambda)$ is predicted to be equal to unity. The effects of SU(3) breaking are seen to be very large for $R(p,\Sigma^+,\Xi)$ and very small for $R(\Xi,\Lambda)$. This contrast persists also in the broken SU(6) model[1,3,6] which introduces SU(3) breaking in the quark moments but not in the baryon wave functions. The broken SU(6) model predicts

$$R(p,\Sigma^+,\Xi)_{\text{theo}} = (\mu_p + 3\mu_\Lambda)/\mu_p = 0.34 \pm 0.005 \tag{2a}$$

$$R(\Xi,\Lambda)_{\text{theo}} = (8/9) - \mu_p/27\mu_\Lambda = 1.06 \tag{2b}$$

The agreement between theory and experiment is now excellent for $R(\Xi,\Lambda)$ and terrible for $R(p,\Sigma^+,\Xi)$. This striking contrast gives interesting information about the underlying physics. The quantity $R(p,\Sigma^+,\Xi)$ depends almost entirely on contributions from nonstrange quarks whereas $R(\Xi,\Lambda)$ depends almost entirely on the contributions from strange quarks. The numerator of the relation (1a) is larger than the prediction of the simple model because the nonstrange quarks contribute less in the Σ than in the proton. The denominator of (1a) is smaller than the model prediction because the contribution of the nonstrange quarks in the Σ is smaller than in the proton. The stability of the expression (1b) against SU(3) symmetry breaking suggests that the contributions of the strange quarks in the Ξ are not appreciably different from the contribution of the strange quarks in the Λ.

These points can be demonstrated quantitatively by a model-independent analysis. The numerator and denominator of (1a) can be expressed in terms of the individual contributions from each quark flavor;[5,7]

$$\mu_p - \mu_{\Sigma^+} = [d(p) - s(\Sigma^+)] + [u(p) - u(\Sigma^+)] \qquad (3a)$$

$$\mu_{\Xi^0} - \mu_{\Xi^-} = [u(\Xi^0) - d(\Xi^-)] + [s(\Xi^0) - s(\Xi^-)] \qquad (3b)$$

where u(B), d(B) and s(B) denote the total contribution of the u, d and s quarks respectively to the magnetic moment of baryon B. These quantities were introduced by Franklin[5] with a different notation. The relations (3) hold for any model of the nucleon which contains only u and d quarks and includes all effects of arbitrary symmetry breaking and configuration mixings as well as relativistic corrections.

If the Ξ wavefunctions satisfy isospin symmetry,

$$s(\Xi^0) = s(\Xi^-) \tag{4a}$$

$$u(\Xi^0) = (-2 + \varepsilon)d(\Xi^-) \tag{4b}$$

where the parameter ε is introduced to include the cases where the magnetic moments of the u and d quarks are not exactly related by the factor −2 of their charges. Attempts to include additional contributions to the quark magnetic moments of this type[6,18] always assume the quantity ε to be small, of the order of a few percent. It has a negligible effect on our results. Substituting Eqs.(3) and (4) into Eq.(1a) gives

$$R(p,\Sigma^+,\Xi) = \frac{[d(p) - s(\Sigma^+)] + [u(p) - u(\Sigma^+)]}{(1 - \varepsilon/3)d(\Xi^-)} \tag{5}$$

In the usual SU(6) treatments the wavefunctions of the two u quarks in the proton and Σ^+ are assumed to be identical as required by SU(3) and the contribution of the second bracket in the numerator of Eq.(5) vanishes. The breaking of SU(3) is expressed by the difference between the d and s moments in the first bracket. Since the wavefunctions of the d quark in the proton, the s quark in the Σ^+ and the d quark in the Ξ^- are assumed to be the same, the contribution from the first bracket on the right hand side is just the fractional difference between d and s moments given by Eq.(2a) and is approximately 1/3. The strong disagreement by almost an order of magnitude with the experimental result of 2.7 given by Eq.(1a) suggests that the second bracket on the right hand side cannot be zero and that the denominator is smaller than predicted by the standard model. Both the magnetic moment of the u quark in the Σ^+ and the magnetic moment of d quark in the Ξ^- are quenched relative to their values in the proton. The only alternative to this quenching is to make the first bracket large by reversing the sign of $s(\Sigma^+)$ compared to $d(p)$. Such reversal of a quark spin is a very violent violation of SU(3) symmetry which does not seem reasonable in any model.

The necessity for this quenching effect is demonstrated more explicitly with the use of SU(3)-breaking and quenching parameters defined by the relations

$$s(\Sigma^+) = \xi_1 d(p) \tag{6a}$$

$$2s(\Xi) = -\xi_2 u(p) \tag{6b}$$

$$u(\Sigma^+) = q_1 u(p) \tag{6c}$$

$$d(\Xi^-) = q_2 d(p) \tag{6d}$$

The symmetry breaking factors ξ_1 and ξ_2 express the ratio of the strange quark to the nonstrange quark moments, defined to be equal to unity in the SU(3) symmetry limit. In the broken SU(6) models[1]

$$\xi_1 = \xi_2 = -3\mu_\Lambda/\mu_p = 0.66 \pm 0.04 \tag{7}$$

The quenching factors q_1 and q_2 are taken to be unity in most models including broken SU(6). They express the quenching of the nonstrange quark contributions in the Σ and Ξ with respect to their contributions in the nucleon. Substituting Eqs.(3-6) into Eqs.(1) gives:

$$R(p,\Sigma^+,\Xi) = \frac{(1 - \xi_1)}{q_2(1 - \varepsilon/3)} + \frac{u(p)}{d(p)} \cdot \frac{(1 - q_1)}{q_2(1 - \varepsilon/3)} = 2.7 \pm 0.8 \tag{8a}$$

$$R(\Xi,\Lambda) = -\frac{\xi_2\mu_p}{3\mu_\Lambda} \{1 + [\frac{q_2(1 - \varepsilon)}{\xi_2} - 1] \frac{d(p)}{\mu_p}\} = 1.05 \pm 0.04 \tag{8b}$$

Equation (8a) shows explicitly the strong disagreement with experiment when both quenching factors q_1 and q_2 are set equal to unity. If ξ_1 is taken from the Λ moment as in Eq.(7) the prediction (2a) of 0.34 is obtained. A negative or drastically smaller value of ξ_1 seems highly unreasonable. Thus nonzero values of q_1 and q_2 are required by this model-independent analysis of the data and show that the magnetic moments of the nonstrange quarks are quenched in strange particles with respect to their values in the nucleon.

The success of the prediction (2b) for the expression (8b) shows that the symmetry breaking factor ξ_2 is very nearly equal to the broken SU(6) value (7) and that there is no appreciable quenching factor for the strange quark. This relation is highly insensitive to the quenching factor q_2, since the entire term in which q_2 appears contributes only 6% of the prediction (2b).

These results are completely model-independent as long as no other constituents are considered in addition to the three valence quarks. They apply to models with arbitrary symmetry breakings, relativistic corrections and configuration mixings as well as to models with quark-diquark structures. In this connection it should be pointed out that the results from SU(6) wave functions are stable against configuration mixing and large admixtures are needed to obtain appreciable modifications of the SU(6) results. The contributions from mixing are always proportional to the square of the admixed amplitude; there is never any linear term.

The stability of the SU(6) result can be seen by examining the expressions for the magnetic moments of the most general three-quark configurations with zero orbital angular momentum. there are two independent spin couplings, and the magnetic moments for these cases have been shown to be[2]

$$\mu_0 \equiv \mu[(ab)_{S=0};c]_{S=1/2} = \mu_c \tag{9a}$$

$$\mu_1 \equiv \mu[(ab)_{S=1};c]_{S=1/2} = (2/3)(\mu_a + \mu_b) - (1/3)\mu_c \, . \tag{9b}$$

where the three quarks are denoted by a, b and c, and the basic states chosen have the spins of a and b coupled to zero and one respectively. For the baryon octet, a and b are chosen to be the two nonstrange quarks in the Λ and Σ° and to be the two quarks of the same flavor in all other baryons. The conventional broken SU(6) value of the magnetic moment is given by μ_0 for the Λ and by μ_1 for all the others.

If additional SU(6) breaking is introduced by mixing these two configurations, the resulting magnetic moment is simply the weighted mean of the two moments (9)

$$\mu = \cos^2\theta\ \mu_1 + \sin^2\theta\ \mu_0 = \mu_1 - (\mu_1 - \mu_0)\sin^2\theta \tag{10}$$

where $\cos\theta$ and $\sin\theta$ are the amplitudes for the two configurations (9b) and (9a). There is no cross term between the two configurations because the spatial wave functions are orthogonal and the spatial overlap integral vanishes. This can be seen by noting that particles a and b are identical fermions and required by the Pauli principle to be in an antisymmetric state of all degrees of freedom, including space, spin, color and isospin. Since the two configurations (9) have a and b in states with the same permutation symmetry in color and isospin and opposite symmetry in spin, they must have the opposite spatial symmetry in the relative co-ordinate $\vec{r}_a - \vec{r}_b$. If one is spatially symmetric, the other is antisymmetric, and the two are orthogonal.

This analysis applies to any model with no orbital angular momentum including quark-diquark models. Erroneous results can be obtained in the quark-diquark model by failing to require the quark outside of the diquark to satisfy the Pauli principle with the quark in the diquark. Results from the original diquark model of Lichtenberg[19] must be updated to include quark statistics which was then an open problem because the color degree of freedom had not yet been established.

For the case where a and b have the same flavor, Eq.(10) can be rewritten

$$\mu = \mu_1 \left[1 - \frac{\mu_a - \mu_c}{\mu_a - (1/4)\mu_c} \sin^2\theta\right] \tag{11}$$

Eqs.(10) and (11) show that the SU(6) value μ_1 is an extremum and that mixing reduces the absolute magnitude of the moment for all two-flavored baryons except the Ξ^-, which is the only case where $\mu_0/\mu_1 > 1$.

The result that configuration mixing affects magnetic moments only by terms quadratic in the admixed amplitudes is general and applies also to admixtures with orbital angular momentum. The magnetic moment operator vanishes between the s-state SU(6) configuration and all configurations with orbital angular momentum. There are no linear terms in the admixed amplitudes and the SU(6) moment is again an extremum.

In conclusion, present data indicate a serious disagreement with simple quark models for baryon magnetic moments which cannot be fixed up by symmetry breaking, relativistic corrections, configuration mixing or quark-diquark models. Some mechanism for quenching the contributions of the magnetic moments of the nonstrange quarks in hyperons relative to their contributions in the nucleon must be introduced to fit present data. Better measurements of the Σ^- and Σ^+ moments would give additional information on this quenching. They could establish whether the quenching increases with increasing strangeness or is a constant for all hyperons. The pion exchange model[17] suggests that the quenching should be viewed as an enchancement of the moment of nonstrange quarks in the nucleon, since the nucleons are the only baryons where a charged pion current between two quarks can contribute to the static moment. In this case the nonstrange quark contributions to the magnetic moments should be the same in all hyperons.

FOOTNOTES AND REFERENCES

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